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A hump-shaped transitional growth path as a general pattern of long-run development

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Abstract:

Long-run development is considered as the income transition between the traditional and the modern steady state, where the speed of the transition is low near the two steady states but high and volatile in between. Such a transition implies a hump-shaped relation between the level of per capita income and its growth rate. A hump-shaped growth-income path can be simulated with a two-sector growth model, where the traditional sector is gradually replaced by the modern sector. Kernel regressions reveal a noisy but robust hump-shaped relation between the growth rate and the level of per capita income in stacked cross-country panel data.

Keywords: Long-run development, transitional growth, two-sector model, kernel regressions

JEL code: O41, O47, C49

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1. Which model of long-run development?

Most modern macroeconomic textbooks use a cross-country version of the Solow (1956) model to account for the large international differences in per capita income. This workhorse model predicts hyperbolic transitional dynamics: a country further away from the steady state should grow faster than a country close to the steady state. Absolut income convergence would follow as a general pattern of development if all countries were converging to the same steady state.

Missing empirical support for absolute income convergence has led to the concept of "conditional" convergence, where each country converges to its own steady state.[3] Assuming that countries have different steady states implies that there is no general pattern of long-run development. However, it is not clear that long-run development should be modeled as a deviation from a country-specific steady state, as in the workhorse model. Long-run development may alternatively be considered as the transition of income between two common steady states, as in unified growth theory (Galor 2005a) and in an earlier strand of the literature (e.g., Lewis 1954, Rostow 1960, Maddison 2001). The latter claims that long-run development reflects a transition from a traditional to a modern steady state. If the speed of the transition were low near steady states, but high and volatile in between, a noisy hump-shaped relation would follow between the level of per capita income and its growth rate, rather than the hyperbolic relation predicted by the workhorse model.

Ranis and Fei (1961) argue that during the transition from one steady state to another, both may coexist as two sectors, where the traditional sector is gradually replaced by the modern sector. Hence, the growth rate of a poor economy with an initially dominating traditional sector will rise with the growth of its modern sector until the modern sector itself begins to dominate the economy. Thereafter, the transitional growth rate will fall to the growth rate of the modern steady state, which is assumed to be determined by the rate of technological change in the leading economies. This two-sector model has recently been reformalized by Lucas (2009) in order to simulate hump-shaped growth paths over time to account for the delayed growth miracle of some initially poor Southeast Asian countries.

To the best of our knowledge, the two-sector growth model has not been estimated as an alternative to the workhorse model of growth empirics. We employ kernel regressions, which do not impose a functional form on the estimation equation, to identify a possible hump-shaped

3. Conditional convergence has been explained with reference to cross-country differences in structural parameters for preferences and technology (Barro 1991, Mankiw et al. 1992). Despite a voluminous subsequent empirical literature, the robustness of the evidence for conditional convergence is still being debated (Abreu et al. 2005, Hauk and Wacziarg 2009, Barro 2015).

growth-income path in cross-country panel data. As is well known, most of the observed variation in growth rates across countries and over time is likely to be due to random shocks (Easterly et al. 1993). A general hump-shaped pattern of development may have been overlooked because it is buried under extremely noisy data and because the workhorse model of growth empirics does not allow for hump-shaped transitional dynamics.

The next section provides brief notes on selected contributions to the literature on transitional growth. This literature has mainly focused on methodical innovations for estimating the (common) speed of adjustment to country specific steady states rather than on estimating a common pattern of long-run income growth. By contrast, many socio-economic variables show a robust empirical transitions that are correlated with per capita income. This leads to the hypothesis that income should also show a common transition path. Section 3 discusses the formal link between a stylized transition curve and its hump-shaped first derivative. Section 4 simulates humped-shaped growth-income paths based on the two-sector model by Lucas (2009). Section 5 addresses data, samples, and specifications. Section 6 reports kernel regressions for cross-country panel data, which reveal a noisy but statistically significant hump-shaped growth path with rising levels of income. This result survives various robustness tests. Section 7 concludes.

2. Brief notes on the empirics of transitional growth

The traditional steady state as first described by Malthus (1798) has prevailed throughout most of the history of humankind. Average incomes were low, and incremental technology advances only supported a slow increase in population size. The transition to the modern steady state, with a persistent growth rate of per capita income in the range of 1-2%, first took off in a few Western countries and their offshoots about two and a half centuries ago.

Over the last 50 years, selected countries from the European periphery and in East Asia, notably Hong Kong, Singapore, South Korea, and Taiwan, have caught up with the high-income countries due to persistent high growth rates, but these selected success stories hardly qualify as a general pattern of catching up and long-run development. In contrast, it has been claimed that poverty traps may keep countries in a traditional steady state (Aziarides and Stachurski 2005) or that the world income distribution is dominated by two rather isolated peaks for rich and for poor countries (Quah 1996).

One of the first empirical studies in favor of unconditional convergence (Baumol 1986) studied developed countries and found convergence. But DeLong (1988) argued that the convergence result did not generalize to a wider sample. For a broad sample of countries, evidence for the "iron law" (Barro 2015) of a theoretically predicted convergence rate of about 2% has only been found by assuming that countries are converging to different steady states, conditional on differences in factor accumulation and other variables. Research on conditional convergence does not ask whether poor countries are catching up with rich countries, i.e., whether there is a general pattern of long run development, but rather whether there is a common speed of adjustment to country-specific steady states.

The present state of empirical research on transitional growth paths is rather unsatisfactory. Despite many methodical innovations over the last 20 years, there are not even robust estimates of the rate of conditional convergence, which is a concept that is at best weakly related to patterns of long-run development. Moreover, the concept of conditional convergence as such comes close to a tautology by predicting that countries would transition to the same steady state if they were the same, except for their initial income. But the relevant empirical question for an assessment of long-run growth and development is whether countries, despite being different and despite facing different starting conditions, have enough in common to allow for a common transitional growth-income path and hence for a general pattern of long-run development.

Apart from the weak empirical evidence in favor of conditional convergence, the long-run transition of an economy from the traditional to the modern steady state is probably not the same process as the adjustment of an economy that is close to but not in its modern steady state. The latter process motivated the original contribution by Solow (1956). There is no apparent reason why one should impose the log-linear convergence restriction derived from the workhorse model on the specification of a growth equation to be estimated with cross-country panel data. Put differently, the workhorse model is obviously a useful tool for studying the short-run adjustment to the steady state path after an exogenous shock, but it may be the wrong tool for studying the long-run transition between two fundamentally different steady states.

In the 1960s and 70s, development theory explained the transition to a modern steady state with the co-existence of a static traditional sector (agriculture) and a dynamic modern sector (industry). According to this approach, development meant the gradual replacement of the traditional sector by the modern one (Lewis 1954, Ranis and Fei 1961). More recently, Lucas (2009) reintroduced the two-sector model of development in a version that refers to modern growth theory and to empirical evidence on cross-country patterns of catching up and convergence.

Empirical support for a transition between steady states and a corresponding hump-shaped growth path comes from the systematic pattern that can be found for many socio-economic variables. For instance, birth and mortality rates, the share of agriculture in GDP, and the levels of corruption, democracy, and religiosity all have systematic transitions with rising levels of income. The statistical details of the transitions can of course be debated, but their empirical relevance as a general pattern of development over the last two centuries can hardly be disputed.[4] The main finding of this literature is that the transitions are slow at the two ends but fast in between, resulting in a hump-shaped pattern of change of the transition variable. From this finding, it is a small step to speculate that a similar pattern might also hold for income as a transition variable, namely a hump-shaped path of the growth rate.[5]

4. The empirical literature on socio-economic transitions is vast and dates back to the seminal contribution by Kuznets (1966). Recent contributions include, e.g., Galor (2005b) and Herrendorf et al. (2014). Our own contributions deal with the transitions of corruption (Gundlach and Paldam 2009), religiosity (Paldam and Gundlach 2013), and democracy (Paldam and Gundlach (2012, 2018).

5. Laursen and Paldam (1982) is an early empirical study that reports a hump-shaped growth-income path for a limited sample of countries.

3. A stylized transition curve and its first derivative

A transition curve is defined as the long-run path of the variable $z(x)$ from one equilibrium to another, with x as the variable that determines the range of the transition. For instance, the left part of Figure 1 shows a stylized transition curve, which is based on the logit function[6]

(1) $$z(x) = \left(1 + e^{-x}\right)^{-1}.$$

S-shaped functions (sigmoids) like the stylized transition curve have the general property of a hump-shaped first derivative.[7] The first derivative of equation (1) equals

(2) $$z'(x) = z(x)\left(1 - z(x)\right),$$

where the shaded area below the hump-shaped path for z' represents the units of change of z during the transition between the two equilibria. This result visualizes that any transition variable z that can be described by a sigmoid function will have a growth path with a peak between the two equilibria.

Therefore, the growth path of income per capita should have a hump-shaped form if long-run development is considered as a transition from a traditional to a modern steady state. The growth path derived from the workhorse model of growth empirics is different because it is based on a function with strictly diminishing returns. Such a specification implies a hyperbolic growth path. For instance, the (production) function

(3) $$y(x) = x^{1-c},$$

where $y(x)$ is per capita income as a function of aggregate factor input x and c is a constant with $0 < c < 1$, implies the first derivative

(4) $$y'(x) = \left(1 - c\right) x^{-c}.$$

The left part of Figure 2 visualizes the different growth paths that result from the transition curve (1) and the production function (3), which both aim to describe the process of long-run

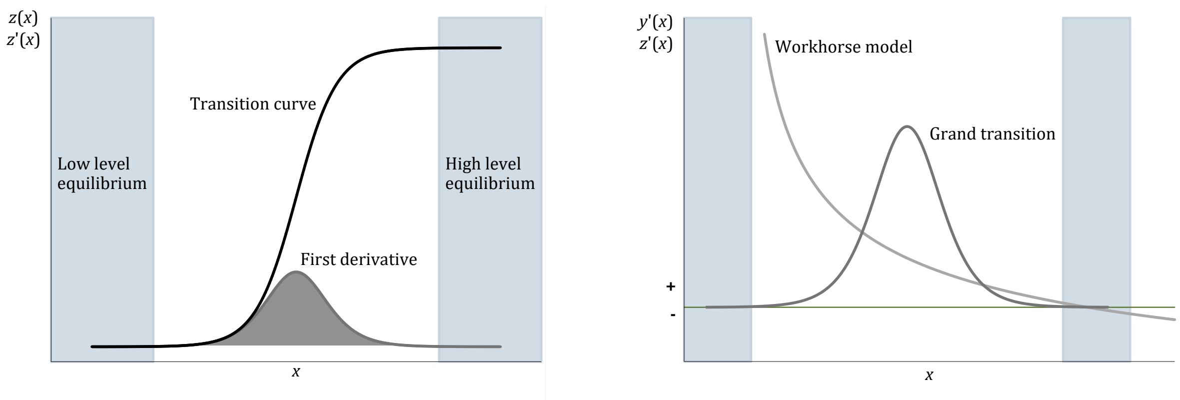
6. The figures in this section are coded as function graphs in Stata; the do file is available on request.
7. See, e.g., Weisstein (undated).

development.[8] Accordingly, "grand transition" (Paldam and Gundlach 2008) refers to the growth path $z'(x)$ implied by a stylized transition of per capita income from a regime with zero steady state growth to a regime with positive steady state growth;[9] "workhorse model" refers to the growth path $y'(x)$ implied by the neoclassical textbook growth model.

The main difference between the growth paths is the prediction with regard to the timing of the growth miracle. The grand transition hypothesis predicts a growth miracle between the low-level (left-hand-side shaded area) and the high-level equilibrium (right-hand-side shaded area). The workhorse model predicts a growth miracle at an early stage of development near the low-level equilibrium and a possible overshooting of the high-level equilibrium with a negative growth rate.

The left part of Figure 2 also reveals that the workhorse model is probably best suited to explain the observed growth fluctuations near the steady state of a mature economy, as intended in the original contribution (Solow 1956). For larger deviations from the steady state, the implied growth path looks implausible: growth miracles have happened in relatively rich countries, after shocks like (world) wars and natural disasters, but not at extremely low levels of income. The hump-shaped growth path implied by the grand transition hypotheses may provide a more realistic account of long run development, at least when supported by an economic model that can generate such a growth path.

Figure 1. Alternative growth paths



8. In Figure 2, $z'(x)$ has been calibrated over a range of x from -9 to 10; $y'(x)$ has been calibrated to derive an intersection point with the horizontal zero line at the high income equilibrium.
9. Income is measured in efficiency units to allow for a zero steady state growth rate in both steady states.

4. Simulating a hump-shaped growth path with a two-sector model

The idea of a hump-shaped growth path has roots in the literature on the two-sector model of development of the 1960s and 70s, as noted above. The formal version of the two-sector model by Lucas (2009) assumes that a large agricultural (traditional) sector, in combination with international and domestic spillovers (externalities), delays the low-level growth miracle that would otherwise emerge from the advantage of backwardness according to standard neoclassical reasoning. Since they lack a sufficient amount of human capital, poor and largely illiterate societies at early stages of development may not be able to implement technology available from more advanced countries. Hence, growth miracles may only occur after a critical amount of human capital has been accumulated in the modern sector of the economy.

4.1. Notes on the formal structure of the Lucas model

The Lucas model has "city" and "farm" as two sectors, which both produce a single output good that adds up to GDP. Cities are held to be the centers of intellectual exchange. The contribution of the city sector to GDP depends on the level of human capital multiplied with its employment share; it is assumed to generate a positive *agglomeration externality* due to the exchange of productive ideas in cities.

In addition, the city sector is assumed to generate a *productivity externality* that spills over to the farm sector, which makes the farm output and its employment share functions of the level of human capital in cities. Assuming mobility of labor across sectors, the model predicts a declining share of farm employment with rising levels of human capital.

Growth enters the two-sector model in the form of catching up with a frontier economy, which is assumed to grow at a constant rate. As in the workhorse model, the income distance to the frontier has a positive effect on the growth rate of the follower economy, but this effect is assumed to be conditioned by an *openness externality*, such that more open follower economies should grow faster than more closed follower economies, all else constant.

Formally, the growth path g_y of an initially poor two-sector economy that transitions to the steady state of the frontier economy can be modeled (Lucas 2009)[10] as

10. See Lucas (2009) for a detailed motivation of the modeling of the two sectors, for the derivation of the functional forms, and for a critical assessment of the parameter values to be used in the simulations. We maintain the original notation.

(5) $$g_y = \mu \left(1 - farm\right)^{\zeta} \left(H / h\right)^{\theta},$$

where μ is the constant growth rate of the frontier economy and *farm* is the employment share of the farm sector. This implies that the first term in parenthesis represents the employment share of the city sector, with the agglomeration externality, ζ. The second term in parentheses represents the income gap with respect to the frontier economy as proxied by the human capital gap of the follower economy, H / h, conditioned by the openness externality, θ.

Solving the output function of the farm sector for its labor input, the employment share of the farm sector is given as

(6) $$farm = \left(\alpha A / h^{1-\xi}\right)^{1/(1-\alpha)},$$

where α is the labor share parameter of the farm sector, A is the (fixed) amount of land per person, ξ is the productivity externality from human capital h in the city sector. Substituting *farm* in equation (5) with equation (6) gives

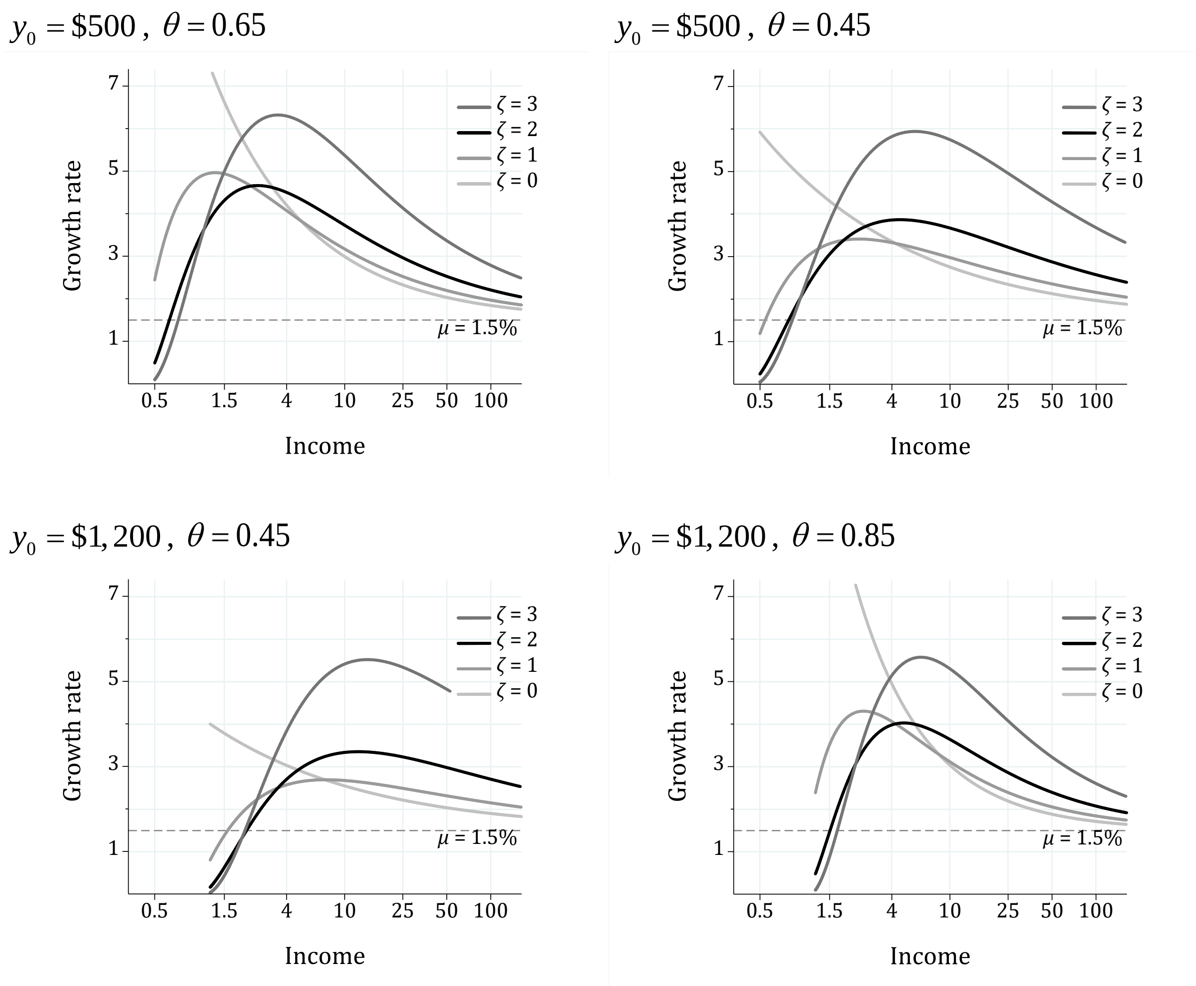
(7) $$g_y = \mu \left[1 - \left(\alpha A / h^{1-\xi}\right)^{1/(1-\alpha)}\right]^{\zeta} \left(H / h\right)^{\theta},$$

which defines the growth path of the two sector model as a function of human capital. Positive growth can only occur if the term in brackets is larger than zero, which is guaranteed by assuming a minimum amount of human capital (i.e., a minimum size of the city sector). For $h \to 0$, the predicted growth rate approaches 0, all else constant. If h is increasing beyond a critical level, where the growth drag of a large farm sector is overcompensated by the effect of the human capital gap, the growth rate will rise until it reaches a maximum. For $h \to \infty$, the term in brackets and the human capital gap term will both approach 1, which implies that after the growth peak, the follower economy will gradually approach the growth rate of the frontier economy.

4.2 Simulating growth paths with the Lucas model

Simulating a growth path with equation (7) requires parameter values for μ, α, A, ξ, ζ, and θ, together with initial values for $y(h)$, $Y(H)$, and *farm*. We largely maintain the initial values and the parameterizations used by Lucas (2009), who focuses on the path of the growth rate over time.[11] Instead, we focus on the path of the growth rate over the level of income, which is a function of *h* according to the underlying production function.[12] In addition, our interpretation of the simulation results differs: Lucas (2009) emphasizes a special pattern for a small group of countries, we emphasize a general but noisy pattern for a broad sample of countries.

Figure 2. Simulated growth paths of the Lucas model



All four panels of Figure 2 show simulated growth-income paths for four values of ζ, ranging from 0 to 3, which are combined with alternative values for the openness externality, θ, and with alternative initial income levels relative to the frontier. The growth rate is measured in percent; income is measured as GDP per capita on a logarithmic scale in $1000. The simulations

11. See Table A1 in the appendix for a detailed comparison.
12. Our simulations are coded in Stata; the do file is available on request. The code can also be used for an exact replication of the simulated growth paths over time reported by Lucas (2009).

are run over 300 periods. To maintain the same scale of the axes of the four panels, growth rates are capped at 7.4% and income is cut off beyond a level of about $165,000.

If the agglomeration externality is set to $\zeta = 0$, the growth path of the (one-sector) workhorse model is replicated. Setting $\zeta > 0$ implies a two-sector model, which generates a hump-shaped growth path by construction. A higher value of ζ has three effects: a lower initial growth rate, a higher the peak growth rate, and a higher the income level at which peak growth occurs. The four panels show how alternative parameterizations of the initial income level and the openness externality (θ) affect the shape and the timing of the hump for each of the alternative values of ζ.

Relative to the top-left panel, a lower value of θ reduces peak growth, flattens the slopes of the hump, and shifts the hump to a higher level of income (top right panel). Starting at a higher initial income level (of 10% of the frontier economy) with the lower value of θ strengthens the three effects (bottom left panel), while a higher value of θ reverses them (bottom right panel). Holding constant the value of ζ (black line), the model always predicts a hump, as it must, but cannot precisely identify the size, the shape, and the timing of the hump without narrow empirical estimates of ζ and θ, which are missing.

Taken together, the simulations show that a two-sector model can account for a broad range of hump-shaped growth-income paths. Even for the limited range of parameterizations reported in Figure 2, the simulated growth peaks occur over a relatively broad income range between about $1,500 and about $15,000 and the peak growth rates vary from less than 3% to more than 6%; depending on the assumed externalities of openness and agglomeration. These externalities may differ across countries if they reflect the impact of country-specific policies that hinder or foster economic growth. Drawing all the alternative simulated growth paths of Figure 2 into a single graph would still give an averaged hump-shaped relation between the growth rate and the level of income, but a rather noisy one. This is probably what one could expect to find in stacked cross-country panel data.

5. Data, samples, and kernel specifications

5.1 The Maddison Project Database

The Maddison Project Database (2018) is used to estimate the relation between the annual growth rate of a country (g_{it}) and its logarithmic income level at the beginning of period *t*, which replicates the setting used for the simulations in the previous section. Income is defined as real GDP per capita in constant (2011) US$ (multiple benchmarks). The time dimension is 1950-2016, which allows for a fairly balanced country-year sample. The included countries account for more than 95% of the world population.

A special feature of the Maddison data is that the income series may not be limited to the years of political independence of a country. For instance, income data are included for Sub-Saharan African countries since 1950, when all but three were colonies. Income data are also included for the successor states of Yugoslavia, the USSR, and Czechoslovakia before splitting off in 1990; for the latter three, income data are continued after splitting as well. We use the available income data for as many countries as possible but exclude forward or backward overlapping observations between existing states and their successors.

Moreover, we delete outliers, which we define as all observations in the first and in the 99th percentiles of the growth rates and the income levels (398 observations, less than 4% of our sample). This procedure eliminates from the sample annual growth rates lager than 26% and smaller than -21% and income levels below $610 and above $57,250.

Extreme negative growth rates appear to reflect (civil) wars, failed states, the breakdown of the Soviet Union, or large negative changes in oil prices. Extreme positive growth rates appear to reflect the exploitation of newly discovered natural resources and mean reversion after negative shocks. An income level of $610 is compatible with the lowest income levels that are recorded in the Maddison Project Database for pre-industrial times: the first percentile of GDP per capita is at $650 for all countries and all years before 1750. Hence still lower income levels can be considered as extreme by historical comparison. Income levels beyond the 99th percentile are also extreme, because they do not exclude observations for countries as rich as the United States. Extreme high income observations are dominated by so-called oil countries.

The latter is the reason why we also eliminate from the sample countries that may have reached high levels of income from exploiting a natural resource rent without going through the grand transition. Especially countries with oil production as the dominant industry often maintain a social, political, and cultural structure that is typical for low income levels and not compatible with a modern steady state. We use OPEC membership in any year before 2019 to identify the resource-rich countries that are deleted from our sample (794 additional observations deleted). OPEC membership is certainly a rather crude proxy to identify non-transition countries, but our results in the robustness section below show that the exclusion of this country group does not affect our main result. Table 1 provides summary statistics for our alternative samples.

Table 1. Descriptive statistics for alternative samples, 1950-2016

	All observations (*ALL*)		*ALL* without extreme observations (*BASIC*)		*BASIC* without OPEC members (*MAIN*)	
	Growth rate g_{it}	Income y_{it-1}	Growth rate g_{it}	Income y_{it-1}	Growth rate g_{it}	Income y_{it-1}
Mean	0.025	9,401	0.025	8,666	0.025	8,563
Standard dev.	0.082	13,292	0.064	9,971	0.058	9,935
Maximum	1.336	220,717	0.260	56,319	0.260	56,319
Minimum	-0.629	134	-0.210	609	-0.210	609
1st percentile	-0.210	608	-0.148	701	-0.145	695
50th percentile	0.025	4592	0.025	4,580	0.025	4381
99th percentile	0.260	57,244	0.196	44,308	0.189	44,281
Observations	10,329		9,931		9,137	
Years	66		66		66	
Countries	169		168		153	
Min. number of countries per year	139		130		119	

Note: Income is real GDP per capita in 2011 US$ (multiple benchmarks), taken from Maddison Project Database (2018). The growth rate is calculated as the proportional increase of real GDP per capita in year *t* compared to year *t*-1. Extreme observations defined by 1st and 99th percentiles. OPEC members defined as countries that have been an OPEC member in any year before 2019. *ALL* includes all observations; *BASIC* excludes outliers; *MAIN* excludes outliers and OPEC members.

5.2 Kernel regressions

Kernel regressions provide a data-driven identification of the growth-income path without imposing a functional form on the estimation equation. The kernel is a local polynomial smoothing process over all pairwise observations of g_{it} and y_{it-1}, which are stacked and sorted by income.

In Stata, kernel regressions require three choices: the bandwidth, the kernel function and the degree of the polynomial smooth. With more than 9,000 observations in our main sample, the kernel is robust to the choice of the kernel function, but it remains sensitive to the bandwidth (see section 6.4). The Stata defaults are the Epanechnikov kernel and a polynomial smooth of degree zero. These choices generate a rule-of-thumb bandwidth, which will change conditional on the selected sample. We also experiment with a fixed bandwidth of 0.31 (see section 6.3), which is close to the rule-of-thumb bandwidth of our main sample.

The Kernel regressions come with confidence intervals around the estimated growth-income path, which reveals the empirical relevance of alternative hypotheses on long-run development. For instance, the confidence intervals may be narrow enough (at conventional levels of statistical significance) to reject the hypothesis of hyperbolic catching up, namely a growth path with persistent negative slope, in favor of the hypothesis of no catching up at all, namely a flat growth path. Or they may be so narrow that our grand transition hypothesis, namely a hump-shaped growth path, cannot be rejected.

However, the kernel regressions face two related problematic features. One is that the cross-section and the time dimensions of the data are merged, which implicitly assumes similar cross-country and the time-series effects of income. If they differ, any reported result may just be a figment of the underlying data without relevance for individual countries. At the same time, it is difficult to interpret multidimensional kernels, which have been proposed to control for the potential effects of omitted variables on the dependent variable. Hence, our kernel estimates of the bilateral relation between g_{it} and y_{it-1} can only identify a limited part of the overall variation in growth rates.[13]

We address the related problems in two ways. First, we estimate panel regressions of g_{it} on y_{it-1} and a quadratic income term y_{it-1}^2, where the latter is included to allow for a hump-shaped growth path and fixed effects are included to control for omitted variables. Second, we estimate growth paths with kernel regressions for individual countries with income data for more than 100 years in Maddison Project Database (2018).

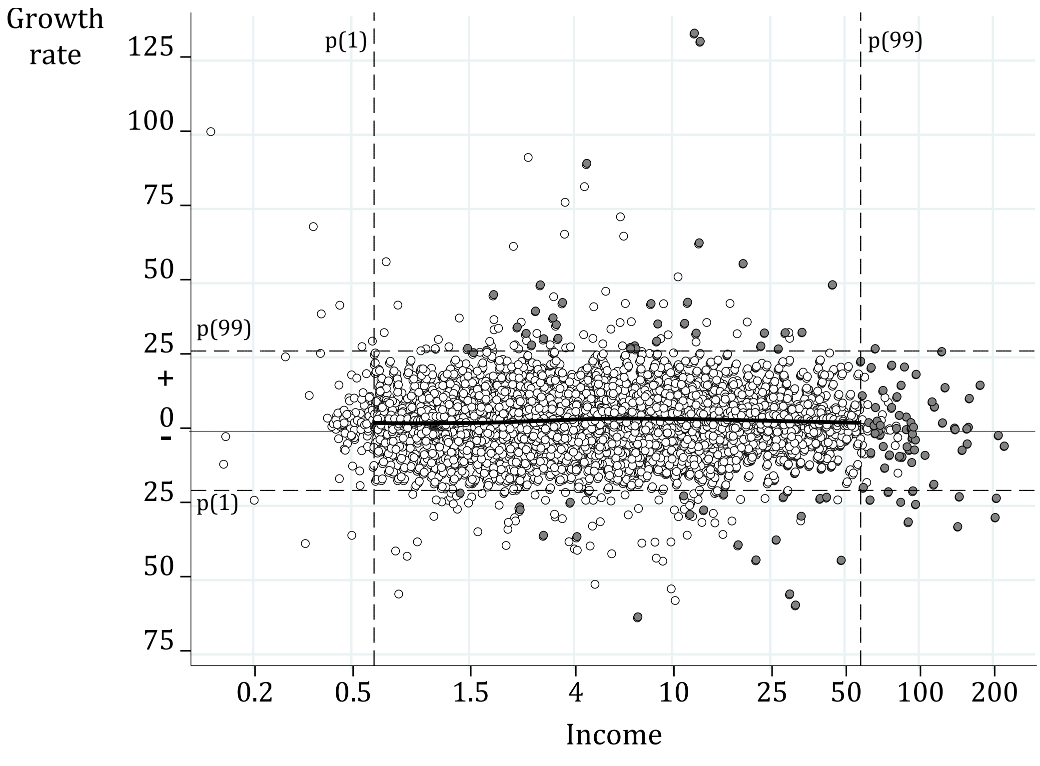
13. See Murtin and Wacziarg (2014) on the interpretation of multidimensional kernels.

6. Empirical estimates of the growth-income path

6.1 *Main result based on a kernel regression*

Figure 3 shows a scatter plot of the correlation between the growth rate and the income level. Each dot represents one of the 10329 country-year observations for pairs of g_{it} and y_{it-1} in 1950-2016.[14] The dashed vertical and horizontal lines identify observations above and below the first and the 99th percentiles of the growth rate and the income level. Shaded dots represent observations for OPEC countries, which account for a large fraction of the outliers. The inner rectangle gives the observations for the samples used below.

Figure 3. The growth-income scatter for the full sample, 1950-2016



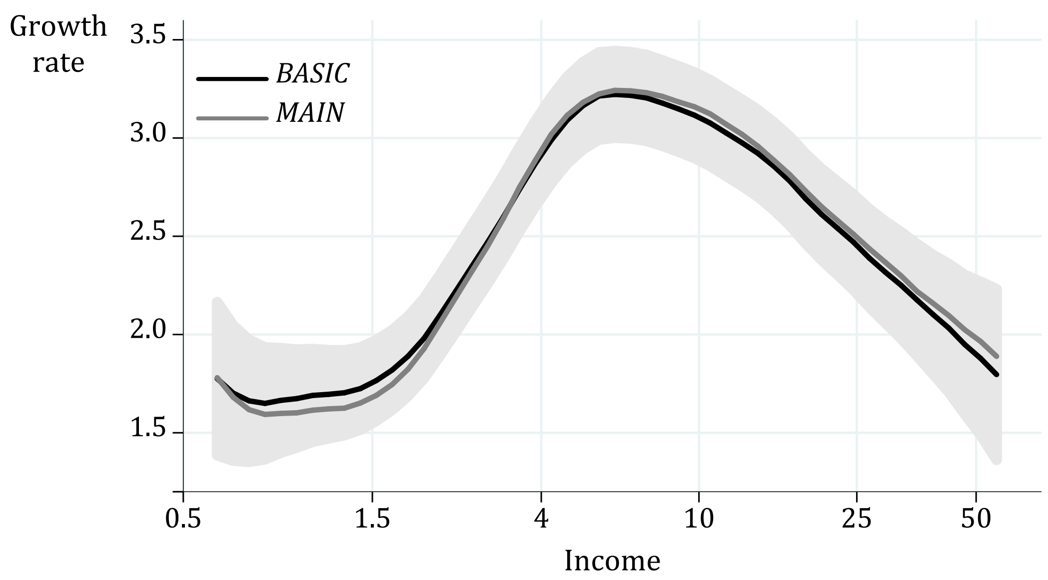
The wild scatter of the data points and the packed rectangle means that any presumed growth path can only explain a small fraction of the variation at best. The black line through the middle of the inner scatter represents the kernel regression estimate of the growth-income path for the

14. For definitions of variables used in Figure 3 and all other figures, see Table 1.

sample without outliers (*BASIC*). With a range of the vertical axis (annual growth rate) from +125% to -75%, the reported kernel line looks flat.

Figure 4 gives an enlarged picture of the same growth-income path, where the scatter points are suppressed. The scale of the income axis is reduced to the range between the first and the 99th percentile; the scale of the growth axis is reduced to a range of a little more than one percentage point around the reported mean growth rate of 2.5%. Zooming in reveals a hump-shaped growth path that picks up at an income level of about $1,500, peaks with a growth rate above 3% at an income level of about $8,000, and thereafter falls toward a potential steady state growth rate slightly below 2%.

Figure 4. The enlarged growth-income path for restricted samples



The graph also shows that it does not matter for the estimated growth path whether OPEC members remain in the sample or not as long as extreme observations are deleted. There is an almost perfect overlap between the paths estimated on the *BASIC* sample (black line) and the *MAIN* sample (gray line). The estimated rule-of-thumb bandwidth (0.32 vs. 0.31) is close, and the confidence intervals are so similar that only one is shown (for *BASIC*) as a shaded area in order to avoid a complete overlaying of the confidence interval of the alternative sample. Unless stated otherwise, all further estimates are based on the slightly less noisy *MAIN* sample (no outliers, no OPEC members).[15]

The reported growth path looks similar to the simulated growth paths for individual economies with alternative initial incomes and alternative parameter values for openness and agglomeration externalities (see section 4). The reported confidence interval is sufficiently narrow to rule out a flat path, which would imply that there is no systematic relation at all between the growth rate and the level of income. It is also impossible to draw a straight line with negative slope within the confidence intervals over the full income range, as implied by the hypothesis of hyperbolic catching up. Hence our main empirical result suggests that a hump-shaped growth-income path cannot be rejected for samples that restrict the variation in growth rates and income levels to the range between the first and the 99th percentile.

6.2. Non-linear panel regressions with fixed effects

The kernel regression that generates our main empirical result ignores omitted variables and the panel structure of the data. To address both concerns, we approximate the hump-shaped growth path with panel regressions that include country- and time-fixed effects together with a quadratic income term to allow for non-linear effects. We check if the marginal income effects change from positive to negative with rising levels of income, as predicted by the kernel regressions. In all regressions, the statistically significant coefficients are positive to income and negative to squared income. But the explanatory power is low, as expected.

Column (1) of Table 2 gives the results for Pooled OLS, which serve as a point of reference. We consider marginal effects at income levels that can be directly compared with the income levels in Figure 4. The marginal income effects change as predicted by the kernel regression: positive at low-income levels and negative at high-income levels, and larger in absolute value at both ends (at $1,500 and $25,000) than near the peak of the hump (between $4,000 and $25,000). The negative coefficient of 0.008 at the high income end implies a rate of convergence of about 1%.[16]

Column (2) reports results for the inclusion of time-fixed effects, which eliminates from the sample the effects of common shocks but retains the cross-country variation. Like Pooled OLS, this specification produces a reasonable approximation of the growth path identified by the kernel regression: the marginal effects are estimated to be statistically significantly different

15. In section 6.4, we show that the growth-income path looks different for a sample of OPEC countries.

16. The exact convergence rate (λ) can be calculated from the estimated regression coefficient (b) as $\lambda = -\ln(1+b)/t$, with $t = 1$ for annual growth rates.

from zero and have the right signs and relative sizes for both sides of the hump. The implied convergence rate at the high-income level is about 1.5% but not much larger than the implicit divergence rate at the low-income level, which implies a net convergence rate close to zero.

The results change with the introduction of country-fixed effects in column (3). Eliminating the cross-country variation from the sample is like assuming that all countries are the same except for their income level, so it is almost by default that the statistically significant marginal income effects are all estimated to be negative. At the high-income level, the negative coefficient of -0.02 implies a convergence rate of about 2%, which is in line with results reported in the conditional convergence literature noted above. Column (4) reports results for the inclusion of both country- and time-fixed effects. Not surprisingly, the marginal effects are much like the marginal effects estimated with country-fixed effects only.

Taken together, the results in Table 2 confirm the hump-shaped growth path of Figure 4 if the cross-country variation is maintained (columns (1) and (2)) and they confirm the results of the conditional convergence literature if it is eliminated (columns (3) and (4)). Not controlling for obvious cross-country differences, except for the level of income, as in the first two specifications, will necessarily produce an omitted variables bias. But eliminating all cross-country variation, as in the latter two specifications, may be too much of a good thing, especially when assessing a potential pattern of long-run growth and development. After all, the long-run information appears to be in the cross-country variation of income levels, not in within-country variation of growth rates over time.[17]

The grand transition from the traditional to the modern steady state is a hypothesis that relies on cross-country and on time series evidence. Treating the cross-country variation as a source of omitted variables bias must lead to a rejection of the grand transition hypothesis for the sample at hand, because the within variation of growth rates in 1950-2010 does not suffice to capture the transition from a static to a modern steady state for individual countries. Maintaining the cross-country variation helps to identify a hump-shaped growth-income path with kernel and panel regressions. This is not to deny that the level of income only explains a tiny fraction of the observed variation in growth rates across countries and over time, but ignoring the grand transition pattern means missing a signal in the noise.

17. Hall and Jones (1999) use this argument to motivate their cross-country regressions on the effect of institutions on long-run economic performance. Along the same lines, Frankel and Romer (1999) use cross-country regressions in *levels* to estimate the effect of trade on (long-run) growth. The combination of persistent country characteristics and non-persistent within-country growth rates, which has been emphasized by Easterly *et al*. (1993), also speaks against eliminating *all* cross-country variation from the sample, because otherwise nothing but regression to the mean may be left.

Table 2. Non-linear panel regressions

	Dependent variable: annual growth rate			
	(1)	(2)	(3)	(4)
Income	0.0663	0.0890	0.0755	0.0959
	(0.009)	(0.013)	(0.019)	(0.019)
	[0.000]	[0.000]	[0.000]	[0.000]
Income squared	-0.0037	-0.0051	-0.0047	-0.0065
	(0.001)	(0.001)	(0.001)	(0.001)
	[0.000]	[0.000]	[0.000]	[0.000]
Observations	9137	9137	9137	9137
Countries	153	153	153	153
R-squared (adjusted/overall)	0.01	0.07	0.00	0.02
Country fixed effects	no	no	yes	yes
Time fixed effects	no	yes	no	yes
Marginal income effects at:				
$1,500 (7.3 log points)	0.0125	0.0146	0.0063	0.0015
	(0.001)	(0.002)	(0.004)	(0.004)
	[0.000]	[0.000]	[0.090]	[0.676]
$4,000 (8.3 log points)	0.0053	0.0046	-0.0030	-0.0112
	(0.001)	(0.001)	(0.002)	(0.002)
	[0.000]	[0.000]	[0.138]	[0.000]
$10,000 9.2 log points)	-0.0015	-0.0047	-0.0117	-0.0230
	(0.001)	(0.001)	(0.002)	(0.003)
	[0.051]	[0.000]	[0.000]	[0.000]
$25,000 (10.1 log points)	-0.0082	-0.0140	-0.0204	-0.0348
	(0.002)	(0.003)	(0.003)	(0.004)
	[0.000]	[0.000]	[0.000]	[0.000]

Note: Cross-country panel data, 1950-2010 (Maddison Project Database 2018). *Main* sample. Income (begin of period) measured in constant international dollars (natural logarithms). Regression constant not reported, robust standard errors in parentheses, *p*-values in brackets.

6.3 Within-country kernel regressions

For a limited number of countries, the Maddison Project Database (2018) includes annual income data for years before 1950. Most but not all of these countries belong to the group of high-income countries. Apart from resource-rich countries, many present high income countries have always been close to the frontier economies, i.e., close to the United Kingdom in the 19^{th} century and close to the United States in the 20^{th} century. This may explain why today's rich countries tend to have long time series statistics. But our focus is on countries that start with a large income gap to the leading economies, as discussed in section 4. Hence, we are looking for countries that did not belong to the club of rich countries in the past but nevertheless have long time series statistics.

The sample selection for the within-country kernel regressions is based on two somewhat arbitrary criteria, namely an income level of less than 20% of the US level in 1913 and a minimum of 90 annual growth-income observations in 1870-2016. To begin with, 1913 is an early year with income data for a relatively large number of countries in the Maddison Project Database. The selected income level of 20% is higher than the starting level of the simulated growth-income paths because some sample countries have annual observations before 1913, so they may have reached a higher income level than, say, 10% if their catching up with the frontier started earlier. At the same time, relatively poor countries in 1913 cannot have started a process of catching up long before 1913, so we limit the time dimension of our sample to 1870, which is another benchmark year in the Maddison Project Database.

Lowering the benchmark income level from 20% to 10% in 1913 would reduce the sample to just two countries, namely Brazil and South Korea. No country in the Maddison Project Database has a relative income below 8.5% of the US level in 1913. Extending the income level beyond 20% in 1913 would mainly include OECD countries plus Argentina and Chile, two resource-rich countries of the time.

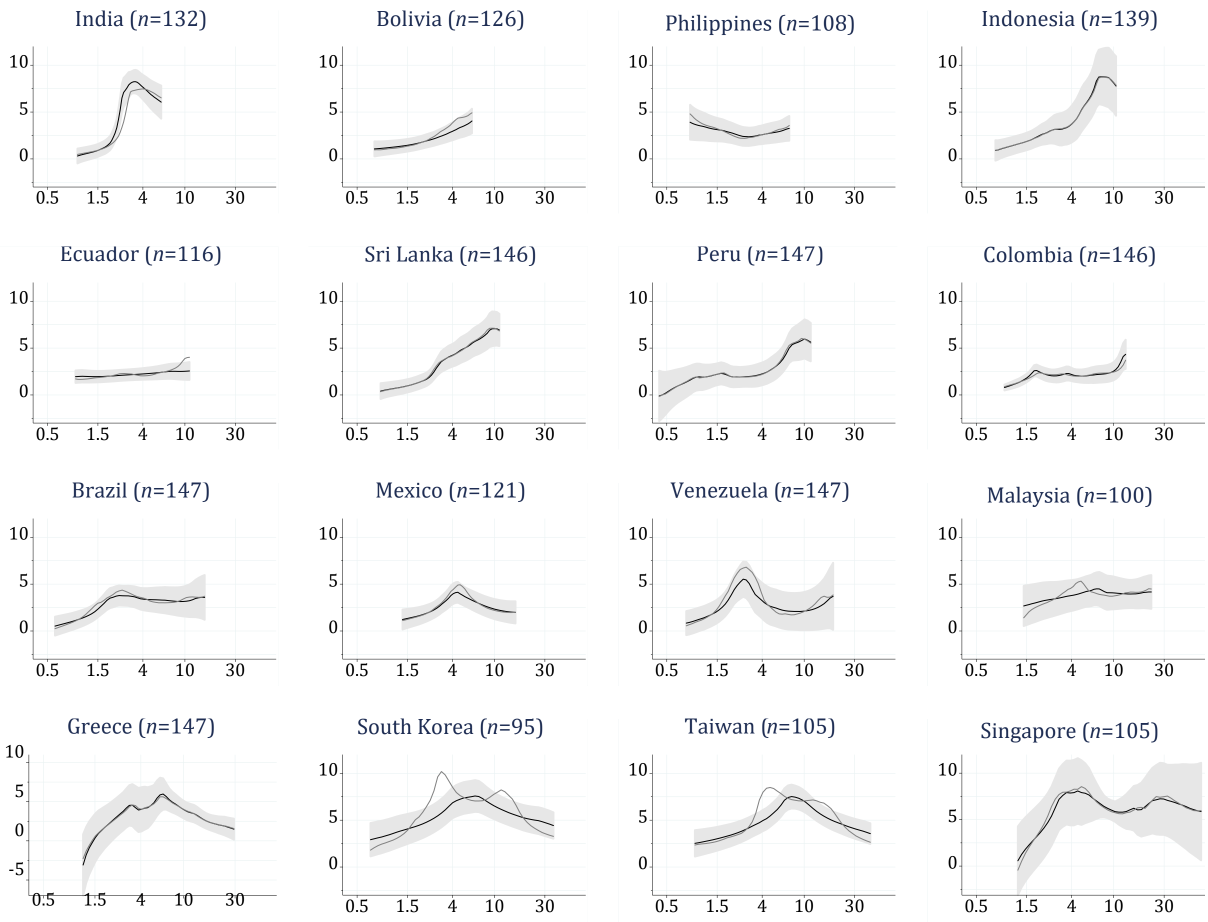
Limiting the sample to countries with more than 90 annual observations is also largely determined by data availability. Reducing the minimum number of annual observations to 70 would include only one additional country (China). Countries with less than 66 annual observations (1951-2016) do not have income data for 1913, which excludes them from the sample. But extending the minimum number of annual observations to 110 would exclude five East Asian countries.

Our sample as defined above includes 17 countries. The number of annual observations varies between 95 (South Korea) and 147 (Greece and four Latin American countries). The country-kernel regressions are presented in Figure 5.[18] The countries are ordered by their highest income level. For each country, the figure shows the rule-of-thumb kernel line selected by Stata (in black) with a 10% confidence interval, together with a kernel line with a fixed bandwidth (in gray) of 0.31, which is the corresponding rule-of-thumb bandwidth of the kernel

18. Romania is excluded because the confidence interval of the kernel line exceeds the 20% growth line.

line in Figure 4 (*MAIN* sample). For most cases, the difference is barely visible. If the difference is visible, it does not change the general shape of the growth-income path.

Figure 5. Growth-income paths for selected countries



Note: Vertical axis: annual growth rate of income (percent). Horizontal axis: income per capita in constant international dollars (in 1000, log scale). *n*: number of annual observations. Countries are sorted by the highest income level. For Greece, the scaling of the y-axis is adjusted.

The growth-income path for individual countries obviously depends on many factors. If the attraction to the frontier described by the model of section 4 is one of them, the within-country kernel regressions should reveal a positive slope of the growth-income path from low to middle income levels and a negative slope thereafter. Our main result (Figure 4) and our panel regressions (Table 2) suggest a growth peak between income levels of about $4,000 and $10,000. This reasoning is confirmed for some countries in Figure 5, but not for all.

For instance, the 10% confidence intervals are so wide for the Philippines and Malaysia that the hypothesis of a flat kernel line cannot be rejected. For Ecuador and Colombia, the narrow confidence interval confirms a flat kernel line. For Venezuela and Singapore, the confidence intervals becomes too wide over parts of the income range to allow for a clear assessment. For the other ten countries, the hypothesis cannot be rejected that there is a positive slope over the lower part of the income range, between about $1,500 and $10,000. For four cases with relatively high income levels, a full hump like the one in Figure 4 becomes visible. We do not find cases with an initial growth miracle or with a growth miracle at high income levels. So the within-country kernel results are not in conflict with our hypothesis that long-run development typically begins with a positive slope of the growth-income path, which is often buried under extremely noisy data.

6.4 Further robustness tests

The panel regressions and the within-country kernel regressions tend to support a hump-shaped growth-income path. In the following, we briefly summarize further robustness tests. The corresponding figures are included in Figure A1 in the appendix. In all cases, the point of reference is the 95% confidence interval of the hump-shaped kernel line estimated on annual growth rates for the *MAIN* sample. Taken together, the kernel regressions for alternative growth rates, bandwidths, time periods, cross sections, and regime types do not show substantial deviations from our main empirical result. However, the growth-income path for OPEC members differs from the stylized pattern.

The left figure in the top row of Figure A1 shows the 95% confidence intervals of kernel regressions with 5-year and 10-year spaced growth rates. As compared to annual growth rates, the confidence intervals for the two averaged growth rates widen with the reduced number of observations and the respective growth peaks slightly decline. However, the hump-shaped form of the growth-income path remains statistically significant for all selected growth rates.

The right figure in the top row demonstrates how alternative bandwidths affect the estimated kernel line (annual growth rates). The shaded area reproduces the 95% confidence interval estimated with the rule-of-thumb bandwidth selected by Stata (0.31). The kernel line becomes quite wobbly for a low bandwidth of 0.1 and approaches a straight line for a high bandwidth of 0.9. In between, the hump-shaped growth-income path remains. For instance, the kernel line remains almost completely within the reference confidence interval for a bandwidth of 0.5.

Deleting alternative decades from the sample also does not change the hump-shaped growth-income path. The only visible effects are minor changes in the level of the growth peak (left figure in the second row). The highest growth peak results if the 1970s and 1980s are excluded, when the world economy first suffered from oil price hikes and later from a debt crisis. By contrast, the lowest growth peak results if the boom decades of the 1950s and 1960s are excluded, when the relatively advanced economies of the time recovered from the losses of the Second World War and enjoyed the (temporary) gains from expansionary economic policies. If the decades after 1990 are excluded, some high income observations are lost but the growth-income path remains within the reference confidence interval of the full sample.

A similar pattern emerges if alternative groups of countries are deleted from the sample (right figure in the second row). To do so, we use regions as coded by the World Bank (World Development Indicators). For instance, *Western Europe and North America* (WE&NA)[19] is dominated by high income observations, so one might expect that the downward-sloping part of the reference confidence interval cannot be replicated if this group of countries is excluded from the sample. Apparently, this is not the case: the kernel line for *excluded* WE&NA remains completely within the reference confidence interval.

Excluding either *Latin America and the Caribbean* (LA&C) or *Asia* (ASIA) from the sample also does not produce substantial differences as compared to the reference confidence interval. Excluding LA&C implies a slightly higher growth peak and excluding ASIA implies a slightly lower growth peak, which is in line with the different average growth performances of the two regions.

Excluding Sub-Saharan African countries (SSA) from the sample gives a different picture. The SSA region accounts for a large fraction of the relatively poor sample countries. Once they are excluded, the growth-income path for the remaining sample countries already displays a positive slope before an income level of $1,500. The SSA region appears to experience a delayed process of long-run development as compared to all other regions.

The left figure in the last row of Figure A1 compares growth-income paths for alternative political regimes. To distinguish between democracies and non-democracies, we use the dichotomous measure coded by Cheibub et al. (2010). The growth-income path for democracies does not differ substantially from the reference confidence interval. Since most high-income countries (apart from some OPEC members) are democracies, the confidence interval of the kernel line for non-democracies becomes rather wide beyond income levels of about $10,000 and extremely wide after income levels of about $20,000 (widening confidence interval not shown). So the reported kernel line for non-democracies is based on a sample that excludes two relatively rich non-OPEC oil countries (Bahrain and Oman) and Singapore, which

19. Our coding of WE is based on the WDI code "ECS"; but we exclude Central Asian countries and formerly socialist countries. ASIA includes the WDI codes "EAS" and "SAS". Details are available on request.

is the only non-oil high-income country that is not a democracy. As it seems, the growth-income path is flatter for non-democracies, but otherwise also does not differ substantially from the reference confidence interval.

Finally, the right figure in the last row compares our main result with the growth-income path for the full *OPEC* sample, which includes extreme growth rates and income levels higher than $60,000. A few OPEC members, and especially Qatar, have even had income levels beyond $100,000 for some years. With very few observations in the income range beyond $100,000 (see Figure 3), the confidence interval of the estimated OPEC kernel line explodes, so we have limited the income range at the high end.[20] Apart from the much wider confidence interval (not shown), the estimated OPEC kernel line differs from our main empirical result. Instead of a hump, the OPEC growth-income path tends to fall throughout, which is a pattern consistent with a return to the steady state after a shock (like finding oil). Hence it is tempting to conclude that resource-intensive countries follow a pattern of development that is different from the grand transition. For all other countries, long-run development is probably better described by a hump-shaped growth-income path.

20. With the income range limited to $100,000, the full OPEC sample (n=920) declines by 21 observations.

7. Conclusion

Kernel regressions based on cross-country panel data reveal a hump-shaped growth-income path. This empirical results contrast with the prediction of a hyperbolic growth-income path derived from the workhorse model of growth empirics. Our kernel regression results suggest that understanding long-run development calls for a two-sector model that can generate a hump-shaped growth-income path.

Our simulation results show that hump-shaped growth-income paths can be generated with a rather broad range of parameters and initial conditions, which determine the timing and the size of the hump. With obvious variation in initial conditions and possible variation of parameters across countries and over time, it becomes understandable why it has been difficult to identify a common pattern of long-run growth and development, especially with a model that imposes the restriction of a hyperbolic growth-income path. Our kernel regressions reveal a common pattern of income dynamics that is overlaid by otherwise extremely noisy data, i.e., most of the enormous variation of observed growth rates remains unexplained.

Employing the usual dose of fixed effects, as is standard in the empirical growth literature, risks missing the signal in the noise. Our main empirical result of a hump-shaped growth-income path is supported by a number of robustness tests. If the long-run path of income can be considered as a transition from a traditional to a modern steady state, it follows by implication that the corresponding growth path must be hump-shaped. Such a growth path can be simulated based on a model that includes a traditional and a modern sector. Taken together, we consider a hump-shaped growth-income path as a general pattern of long-run development.

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Appendix

Table A1. Growth simulations: initial conditions and parameterizations

	Notation	*Present paper*	*Lucas* (2009)
Initial conditions			
Income of frontier economy	Y_0	12,000	12,000
Income of follower economy	y_0	500; 1,200	830
Employment share of farm sector	x_0	0.8	0.8
Parameterizations			
Growth rate of frontier economy	μ	0.015	0.02
Labor's share in farm production	α	0.6	0.6
Productivity externality in farm sector	ξ	0.75	0.75
Agglomeration externality	ζ	0, 1, 2, 3	0, 1, 2, 3
Openness externality	θ	0.45, 0.65, 0.85	0.50, 0.65, 0.83

Figure A1. Robustness tests of the main kernel regression result